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Video ①

Date:

"Number Theoretic Function" (Chap-6)

A function whose domain is the set of positive integers is called a number theoretic (or arithmetic) function. Its value may be integral, real or complex.

The function τ and σ :- For each positive integer n, $\tau(n)$ is the number of positive divisors of n and 1 and n and $\sigma(n)$ is the sum of of these divisors. Thus these functions are define :-

$\tau : N \to N$ such that

$\tau(n) = \sum_{\substack{d/n \\ d \geq 1}} 1$ where $\sum_{d/n} 1$ denotes the sum of as many 1's are divisors of n.

and

$\sigma : N \to N$ such that

$\sigma(n) = \sum_{\substack{d/n \\ d > 1}} d$ where $\sum_{d/n} d$ is the sum of positive divisors of n.

Ex :- Find $\tau(15)$ and $\sigma(15)$

Sol :- The divisors of 15 are 1, 3, 5, and 15

Therefore $\tau(15) = \underline{4}$ and

$\sigma(15) = 1+3+5+15$

$= \underline{24}$

Ex :- If p is a prime then find $\tau(p)$ and $\sigma(p)$.

Sol :- prime p has only two factors 1 and p itself. Therefore $\tau(p) = 2$ and $\sigma(p) = p+1$

Theorem 2.

If $n = p_1^{a_1} \cdot p_2^{a_2} \cdots p_k^{a_k}$ is the prime factorization then

(i) $\tau(n) = (a_1+1)(a_2+1) \cdots (a_k+1)$ and

(ii) $\sigma(n) = \left(\dfrac{p_1^{a_1+1} - 1}{p_1 - 1}\right)\left(\dfrac{p_2^{a_2+1} - 1}{p_2 - 1}\right) \cdots \left(\dfrac{p_k^{a_k+1} - 1}{p_k - 1}\right)$

prove urself.

Th 3 :- For any integer $n>1$ $\tau(n)$ is odd iff n is a perfect square

Proof :- Let $n = p_1^{a_1} \cdot p_2^{a_2} \cdots p_k^{a_k}$ be square of an integer

Then $a_1, a_2, \ldots, a_k$ are all even.

Therefore $(a_1+1), (a_2+1), \ldots, (a_k+1)$ are all odd.

Now $\tau(n) = (a_1+1)(a_2+1) \cdots (a_k+1)$
$= $ an odd integer

(product of odd integer is an odd integer)